## The problem being addressed

As originally developed from the LAMS observations, the fit for the PCOR $\Delta p$ was expressed in terms of raw measurements QCF, PSF, Mach(QCF, PSF), ADIFR, and QCR. The advantage is that, by using only raw measurements, the function then does not depend on any other derived parameters. However, a problem arises if the sensitivity coefficients for the radome change, as is apparently the case. Because a likely dependence of $\Delta p$ is on angle of attack, if the value of ADIFR changes for a given angle of attack then the original formula will lead to an incorrect value for $\Delta p$

This suggests reformulating the equation for $\Delta p$ in terms of angle-of-attack $\alpha$ rather than ADIFR and QCR directly. However, then a problem with implementation arises because QCXC, now used to find $\alpha$, depends on $\Delta p$ but a reformulated $\Delta p$ will depend on $\alpha$. This note suggests a complicated formula as a way around that circularity problem.

## The math

The formula for $\Delta p$ developed from fits to the LAMS data was:

$$
\begin{equation*}
\frac{\Delta p}{p}=a_{0}+a_{1} \frac{q_{\mathrm{m}}}{p_{\mathrm{m}}}+a_{2} M^{3}+a_{3} \frac{\Delta p_{\alpha}}{\Delta q_{\mathrm{r}}} \tag{1}
\end{equation*}
$$

with values of the coefficients $\left\{a_{i, i=0-3}\right\}$ respectively $\{-0.00076,0.073,-0.0864,0.0465\}$. The resulting formula for corrected $q_{c}$ (QCFC) is then

$$
\begin{equation*}
q_{c}=q_{m}-\Delta p \tag{2}
\end{equation*}
$$

The formula for $\alpha$ is:

$$
\begin{equation*}
\alpha=b_{0}+\frac{\Delta p_{\alpha}}{q_{c}}\left(b_{1}+b_{2} M\right) \tag{3}
\end{equation*}
$$

where $M$ is evaluated from the corrected measurements $q_{c}$ and $p_{c}$. However, this term is a minor contributor to the equation and $M$ and $q_{c}$ can be calculated from the uncorrected measurements $q_{m}$ and $p_{m}$ with negligible effect on the resulting value of $\Delta p$. The default coefficients from the Processing Algorithms document are $b_{i}=\{4.604,18.67,6.49\}$; for CONTRAST the appropriate coefficients were found to be $\{4.34685,20.10448,1.36492\}$, and still other coefficients apply to DEEPWAVE.

Equation 3 solved for the pressure ratio needed in Eq. 1 is

$$
\begin{equation*}
\frac{\Delta p_{\alpha}}{q_{r}}=\frac{\left(\alpha-b_{0}^{\prime}\right)}{\left(b_{1}^{\prime}+b_{2}^{\prime} M\right)} \frac{q_{c}}{q_{r}} \tag{4}
\end{equation*}
$$

Note re: Calculating the PCOR function
28 August 2014
Page 2
where primes on the coefficients $\left\{b_{i}^{\prime}\right\}$ indicate that these should be the coefficients applicable at the time of the LAMS calibration that determined the coefficients in Eq. 1; i.e., for PREDICT where $\left\{b_{i}^{\prime}\right\}=\{4.604,18.67,6.49\}$. However, $\alpha$ from subsequent flights when the radome sensitivity coefficients might have changed is given by Eq. 3 with new coefficients $\left\{b_{i}\right\}$. Substituting in Eq. 2 and using Eq. 1 results in the following expression for $q_{c}$ :

$$
q_{c}=q_{m}-p_{m}\left(a_{0}+a_{1} \frac{q_{m}}{p_{m}}+a_{2} M^{3}\left(q_{m}, p_{m}\right)+a_{3} \frac{\left(\alpha-b_{0}^{\prime}\right)}{\left(b_{1}^{\prime}+b_{2}^{\prime} M\right)} \frac{q_{c}}{q_{r}}\right)
$$

or

$$
\begin{equation*}
q_{c}=\frac{q_{m}-p_{m}\left(a_{0}+a_{1} \frac{q_{m}}{p_{m}}+a_{2} M^{3}\left(q_{m}, p_{m}\right)\right)}{1+p_{m} a_{3} \frac{\left(\alpha-b_{0}^{\prime}\right)}{\left(b_{1}^{\prime}+b_{2}^{\prime} M\right)} \frac{1}{q_{r}}} \tag{5}
\end{equation*}
$$

where Eq. 3 should be used to evaluate $\alpha$. For this purpose only, $q_{m}$ could be substituted into Eq. 3 with negligible effect on the result obtained from Eq. 5.

## The Circularity Problem

A problem with this approach, encountered as we tried to apply this to CONTRAST data, is that sensitivity coefficients are determined using Eq. 3, which depends on $q_{c}$, but $q_{c}$ is evaluated using Eq. 5 which depends on $\alpha$. For CONTRAST this was resolved by iteration, in which an initial estimate of $\left\{b_{i}\right\}$ was used to evaluate $q_{c}$, then flight data were used to fit for the coefficients in Eq. 3, and this new estimate of $\left\{b_{i}\right\}$ was used to repeat the calculation of $q_{c}$, etc., until $\left\{b_{i}\right\}$ no longer changed.
This is impractical for routine use, so a different approach is needed.

## Suggested Solution

One change that avoids this circular dependence is to express Eq. 3 in terms of $q_{m}(\mathrm{QCF})$ instead of $q_{c}$ (QCXC). In the past, $q_{r}$ (QCR) was used, but this was changed because invalid measurements from the radome, caused by the radome becoming blocked by ice accretion or by freezing of residual water in the lines from the pressure port to the transducer, were much more frequent than from the pitot tube and this caused problems with many derived measurements that depend on pressure. However, it is just as reliable to base the denominator on $q_{m}(\mathrm{QCF})$ :

$$
\begin{equation*}
\alpha=b_{0}^{*}+\frac{\Delta p_{\alpha}}{q_{m}}\left(b_{1}^{*}+b_{2}^{*} M\right) \tag{6}
\end{equation*}
$$

This makes it possible to determine the coefficients $\left\{b_{i}^{*}\right\}$ independent of the result for $\Delta p$ (except through a minor residual dependence of $M$ on $\Delta p$ ), which turned out to be barely detectable.

The procedure recommended for the future, and for reprocessing past projects other than CONTRAST which has already been done as outlined above, is to use these sensitivity coefficients for angle-of-attack with Eq. 6:
pre-SAANGRIA-TEST $\left\{b_{i}\right\}=\{5.5156,19.0686,2.0840\}$
SAANGRIA-TEST and later: $\left\{b_{i}\right\}=\{4.6049,18.4376,6.7546\}$

## Requirements for a PCOR function

Evaluation of $\Delta p=q_{m}-q_{c}$ then requires this input:

1. Raw measurements $p_{m}=\mathrm{PCF}, q_{m}=\mathrm{QCF}, \Delta p_{\alpha}=\mathrm{ADIFR}, q_{r}=\mathrm{QCR}$.
2. PCOR calibration coefficients $\left\{a_{i}\right\}$ as listed above. These are fixed and can be coded into the PCOR function.
3. Current radome sensitivity coefficients $\left\{b_{i}\right\}$. These change and should be passed to the PCOR function.
4. The sensitivity coefficients $\left\{b_{i}^{\prime}\right\}$ at the time the formula for $\Delta p$ was originally determined using LAMS, as given above. These do not change and can be coded into the PCOR function.

## Ad-Hoc Suppression of Flap-Deployment-or-Retraction Errors

Two additional steps are needed to avoid bad values at slow airspeed:

1. Evaluate the denominator in (5) separately. In normal operation, this should be close to 1 , with only a small correction for the last term in the denominator. To avoid a region of singularity, when this denominator falls below 0.85 force it to 0.85 . This only occurs for TAS $<100$, so it won't affect normal research data.
2. With this imposed limit on the denominator, the value of $\Delta p$ determined from (5) using $\Delta p=q_{m}-q_{c}$ will still have undesirable effects at low airspeed. The problem region only occurs for TAS $<100$, and $\mathrm{QCF}<40$ always leads to TAS $<100$, so a way to taper the value of $\Delta p$ for the problem region (to move $\Delta p$ toward zero as the airspeed decreases) is to taper values of the correction $\mathrm{DP}=\Delta p$ for $\mathrm{QCF}<40$ as follows:
```
if(QCF < 40.) {DP *= (QCF/40.)**3}
```

This avoids the creation of unrealistic perturbations during initial climb or final descent, where it appears that deployment or retraction of flaps causes a change in angle-of-attack through a region of singularity and also causes an airflow change not represented by the standard PCOR function. See, for example, CONTRAST flight 1, 17:42:30. This tapering approach is arbitrary but does keep the measurements from looking too bad during these transitions.

## Documentation

This note is PcorCalculation.pdf, generated by PcorCalculation.lyx, both on cooperw@ucar.edu Google Drive, directory "Algorithms",available to anyone within UCAR. Appropriate modifications to Section 4.4 of the document re algorithms, ProcessingAlgorithms.pdf (same location), have also been made to reflect the recommended processing approach described above.

The fits leading to the recommended values of $\left\{b_{i}\right\}$ above were obtained using the R program AKRDcoef.R located in the EOL directory $\sim$ cooperw/RStudio/CONTRAST. For CONTRAST, the program used was TestReprocessing.R in the same directory. Associated processing for vertical wind for CONTRAST was studied and documented using the program CalibrationCONTRAST.Rnw in that same directory; this also generated the memo CalibrationCONTRAST.pdf in Google Drive for cooperw@ucar.edu, directory Algorithms.

For a possible implementation, see ~cooperw/RStudio/CONTRAST/CheckPcor.R or $\sim$ cooperw/RStudio/Ranadu/R/PcorFunction.R.

