Subject: Recommendation: introduce a rate-of-climb variable, replacing VSPD
Al Cooper
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Background

The objective of this recommendation is to obtain an alternative to the estimate of vertical wind now provided by WI. I argue below that WI as it has been calculated in recent projects is flawed and does not provide a basis for the vertical-wind calculation that is an alternate in the case that WIC, calculated from GGVSPD, is not available. In that case, the new variable suggested here can provide an alternative that still will provide a useful measurement of vertical wind. Indeed, it appears to be better than GGVSPD because it provides a good representation of the highest frequencies.

VSPD is the vertical aircraft speed, or rate of climb, provided by the INS. The INS in use on both aircraft uses updating to the pressure altitude to control the known instability that arises from integrating vertical acceleration to get rate-of-climb and altitude. As a result, the rate-of-climb represented by VSPD is adjusted to match a standard atmosphere, and if the atmospheric stratification differs from the standard atmosphere the resulting rate-of-climb will not be the geometric-altitude rate of climb, as needed for the calculation of vertical wind. In a baroclinic region, false vertical motions can be introduced by this updating.

This can be seen best from the hydrostatic equation, expressed as

$$\frac{dz}{dp} = -\frac{R_a T}{pg} \tag{1}$$

Then

$$w_p = \frac{dz}{dt} = -\frac{R_d T}{pg}\frac{dp}{dt} \tag{2}$$

dz/dt is the rate of climb in terms of geometric altitude. For given climb conditions (T, p, and dp/dt), this only matches the climb rate in a standard atmosphere if the temperature equals that of a standard atmosphere, T_s. Otherwise, the error in rate-of-climb w is:

$$\frac{\delta w_p}{w_p} = \frac{R_d}{w_p p g}(T - T_s)\frac{dp}{dt} = \left(\frac{T - T_s}{T}\right)\ . \tag{3}$$

As an extreme example, if the temperature is 20°C different from a standard atmosphere and the climb rate is 10 m/s where the flight temperature is about -70°C, the error is about 1 m/s.

How this affects VSPD depends on the degree of coupling via the barometric updating. As the INS detects changes in vertical acceleration, it changes the vertical velocity of the aircraft according to that acceleration, and this step is not subject to the error introduced by the baro-loop. However, updating then adjusts this to avoid introduction of altitude and vertical-speed errors by nudging

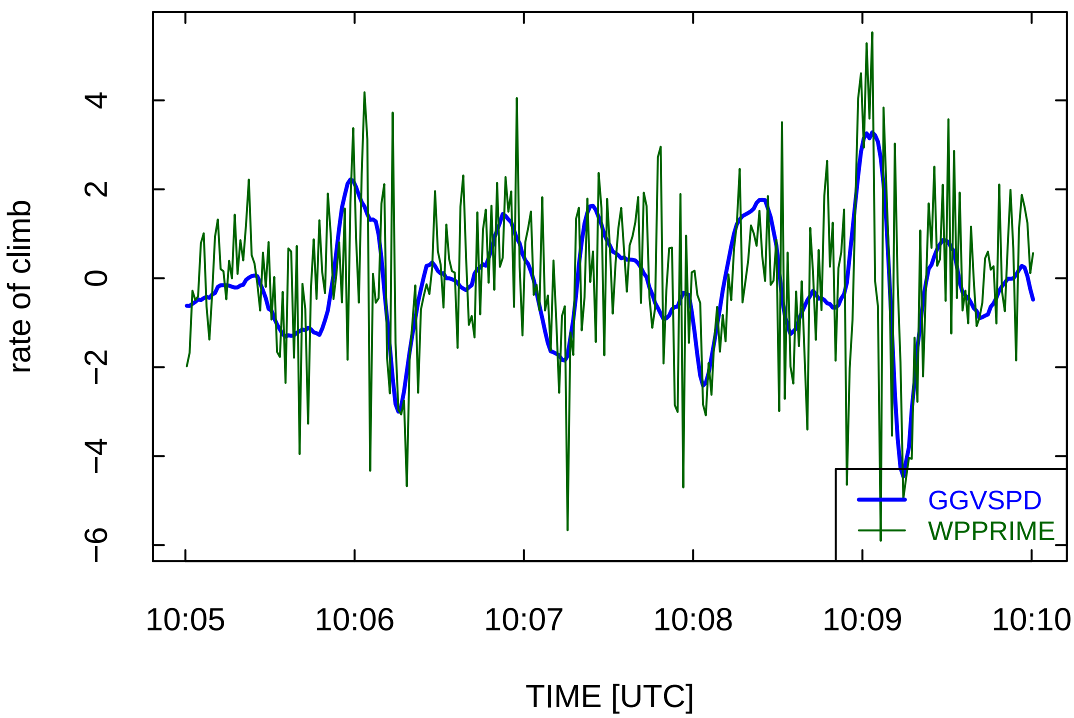

Figure 1: Comparison of GPS-provided rate of climb (GGVSPD) and that calculated from the hydrostatic equation (WPPRIME)

the measurement toward the vertical speed indicated by the rate of pressure change. This causes an error that enters directly into WI when calculated from VSPD, because that updating should be done to a geometric rather than a pressure altitude. The details of the baro-loop and filtering incorporated into the INS are not known to us, so it is difficult to interpret VSPD as needed for calculation of WI.

Suggested Change

Several options are available for improving VSPD or for generating a new variable to substitute for it:

1. Use the time derivative of pressure directly in (2) to find a new variable for the vertical motion of the aircraft. Pressure is now measured with low uncertainty, following the LAMS-based calibration, so this is worth considering. However, Fig. 1 shows that, even at 1-Hz

resolution, the derivative of pressure is too noisy to provide a good representation of rate of climb.

2. Use the IRU-measured vertical acceleration ACINS (or the measured body acceleration vector resolved into the vertical direction) to calculate an alternate estimate of rate-of-climb, and update that variable to match the change in geometric rather than pressure altitude. This could be done by implementing a baro-loop as was previously used for the variable WP3 or alternatively by filtering the estimate of vertical speed obtained from (2) and adjusting the integrated acceleration so that the filtered version of integrated ACINS matches the filtered version of the estimate from (2). This is the approach explored in this memo.

3. Evaluate how VSPD as provided by the INS is adjusted toward pressure altitude, and correct it by separately adjusting toward geometric altitude using (2). This would be complicated because it would be necessary to analyze and reverse-engineer the calculation of VSPD in the INS and that is probably a difficult project.

To pursue the second option, consider these variables: w_p as provided by the right side of (2) and $w_p^* = \int_0^t a(t)dt$ where a is the vertical acceleration ACINS as provided by the INS. Define $\Delta w_p = w_p - w_p^*$ and $\overline{\Delta w_p}$ as a low-pass filtered version of Δw_p. Then estimate the rate of climb of the aircraft from $w_p' = w_p^* + \overline{\Delta w_p}$. The specific implementation of this that was tested in R uses a Butterworth filter to find $\overline{\Delta w_p}$, using:

```
## WP from hydrostatic equation, WPSTAR from ACINS
DPDT <- c(0, diff(PSXC)) * Rate  ## Rate is 25 for high-rate measurements
## Rd <- StandardConstant('Rd'); g <- 9.80 # or Gravity(LAT, GGALT)
WP <- -Rd * (273.15 + ATX) * DPDT / (PSXC * g)  ## P units cancel
WPSTAR <- cumsum(ACINS) / Rate  ## cumsum: running cumulative sum
## fill in missing values by interpolation
DIF <- zoo::na.approx (as.vector(WP - WPSTAR), maxgap=1000, na.rm = FALSE)
ROC <- WPSTAR + signal::filtfilt (signal::butter(3, 2/(tau*Rate)), DIF)
```

where the next-to-last statement removes missing values and replaces them by interpolation, because missing values lead to an error in the Butterworth-filter statement in the last line. As for the standard complementary filter, the last line can be replaced by the filter coefficients generated by the R call and the filtering can be recursive, so this can be incorporated into nimbus. The period of the filter cut-off is tau, here selected after some exploration to be 300 s.

It may be useful to integrate the variable ROC to obtain a new altitude variable that is also independent of the GPS-provided measurement of altitude.

The result of this calculation is shown in Fig. 2 as “ROC”, which is mostly overlapping with GGVSPD in this plot The comparison to GGVSPD is very good for the full flight, with mean difference of -0.08 and standard deviation of 0.17 m/s for the part of the flight that is at least three filter periods from the start and end of the flight. Fig. 3 shows another comparison to the GPS-provided variable GGVSPD and the INS-provided variable VSPD.

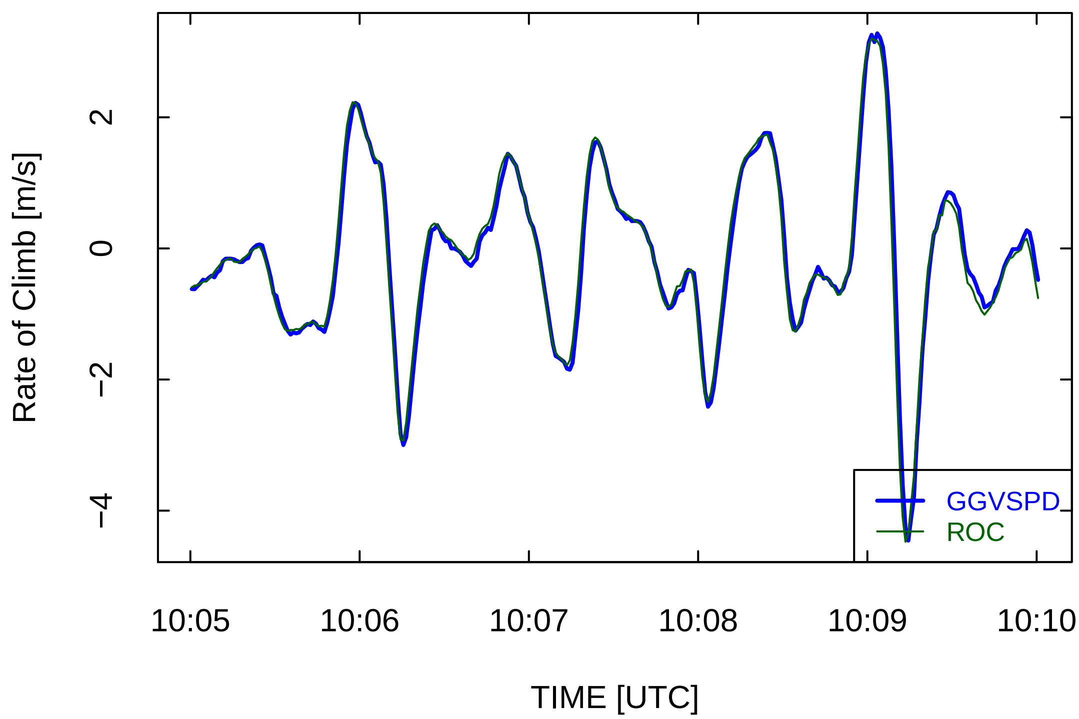

Figure 2: Comparison of GPS-provided rate of climb (GGVSPD) and proposed new variable (ROC)

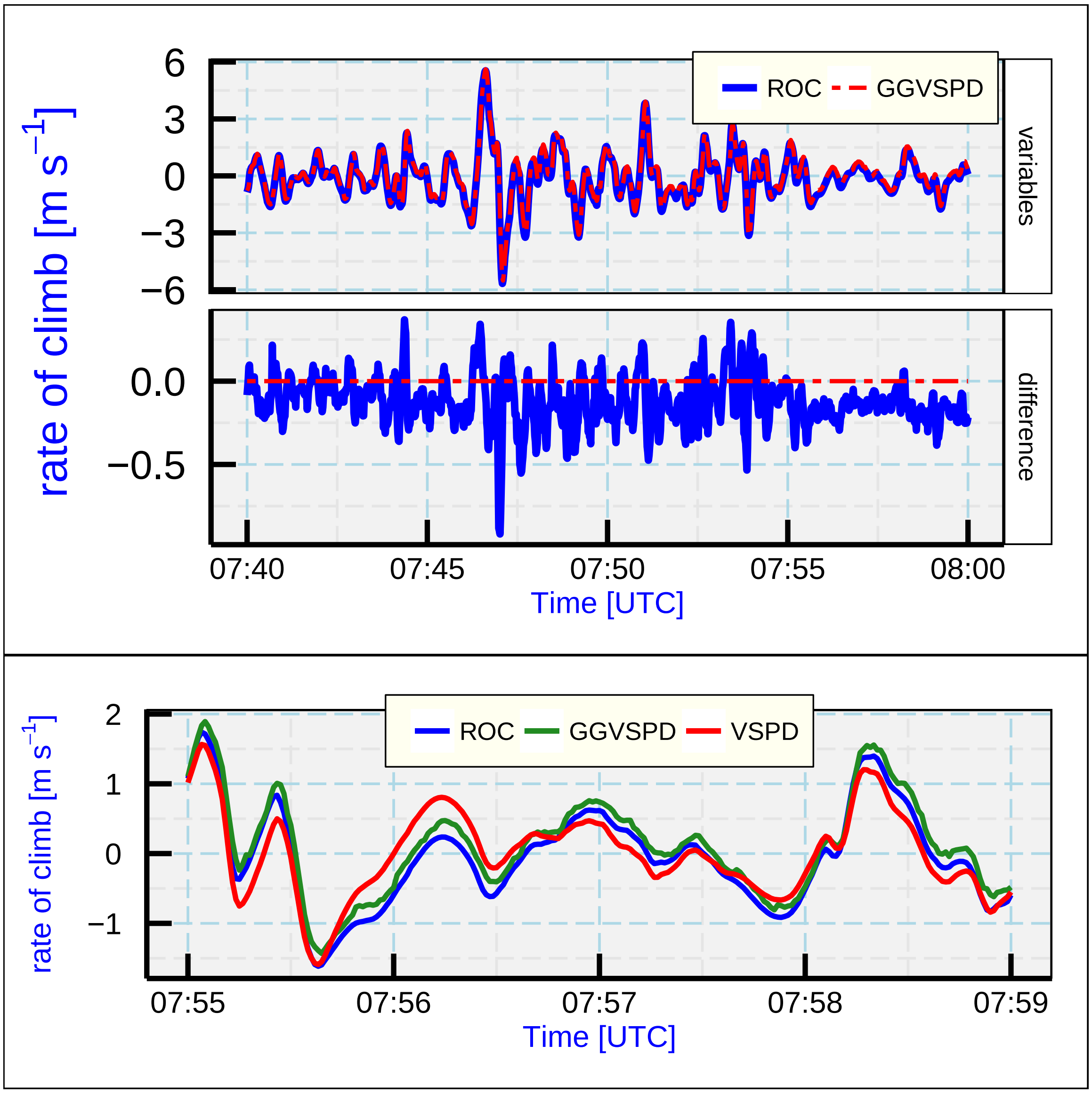

Figure 3: Comparison of new variable representing rate of climb (ROC) to the GPS-provided measurement (GGVSPD) and, in the bottom panel, the measurement provided by the INS (VSPD). The time covered in the bottom panel is a small segment from the time interval used for the top two panels.

Recommendations

1. Create a new variable, perhaps named ROC (rate of climb), calculated using this algorithm (for which reference R code is embedded in the routine generating this memo):

    (a) For each time interval:

    i. Increment a variable WPSTAR that saves the integral of ACINS / RATE where rate is the sample rate. This is the integral of the vertical acceleration and so represents the rate of climb of the aircraft.
    ii. Similarly increment another variable WPPRIME calculated from (2), based on the hydrostatic equation, to obtain a geometric measure of altitude change.
    iii. Update the filtered variable (DIFW) which is the Butterworth low-pass filter applied to WPRIME−WSTAR. This is the same filter now being used for the complementary filter, so the appropriate coefficients are present. (I think the time constant is based on tau=300 s, but if not new coefficients will be needed.)
    iv. Calculate the new variable ROC = WPSTAR + DIFW.

2. Create a backup vertical-wind variable (WIR) based on ROC instead of GGVSPD, using the normal calculation as for WIC but without a lever-arm correction for displacement of the GPS antenna because the measurement is, except for slow updating, already located at the INS and does not reference the GPS measurements. This variable can replace the old calculation of WI, which I think is problematic and should be eliminated from processing.

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