23 September 2015

TO: Reprocessing File
FROM: Al Cooper
SUBJECT: Study of the Recovery Factor

1 Some Relevant Equations

1.1 Basic measurement of ambient temperature

The basic measurement relating the measured recovery temperature T_r to the ambient temperature T_a, with both in absolute units or kelvin, is

$$T_a = T_r - \alpha_r \frac{U_a^2}{2c_p'} \tag{1}$$

where α_r is the recovery factor, U_a the airspeed, and c_p' the specific heat of moist air at constant pressure. According to measurements from wind-tunnels, the recovery factor varies with Mach number. The following equation is now used to represent the recovery factor:

$$\alpha_r(M) = 0.988 + 0.053 \log_{10} M + 0.090 (\log_{10} M)^2 + 0.091 (\log_{10} M)^3 \tag{2}$$

where M is the Mach number.

1.2 Effect of the time response of the sensor

The recovery factor is usually studied with speed runs, where the flight speed of the aircraft is varied from the lowest to highest part of the flight envelope while maintaining altitude. During acceleration, the recovery temperature increases, but the time response of the sensor will cause the measured temperature to lag behind the correct value. Consider the case where the time response is characterized by a simple exponential:

$$\frac{dT_r}{dt} = \frac{T_r^* - T_r}{\tau} \tag{3}$$

where T_r^* is the true value and T_r the sensed value. Then for a steady rate-of-change of true temperature $dT_r^*/dt = a$, to maintain a steady rate-of-change of the sensed temperature requires that $(T_r^* - T_r) = a\tau$. Therefore T_r lags behind T_r^* by $a\tau$ and, for steady acceleration, should be corrected by the addition of $a\tau$ to the measured time sequence. The result would be the same as shifting the time series forward by τ. This remains a good approximation if the rate-of-change of temperature can be considered reasonably linear over the period τ.

For the analysis of speed runs, this suggests shifting the time series to minimize the difference at given airspeed between the accelerating and decelerating branches, and indeed it should be possible to find the time response of the sensor in this way.

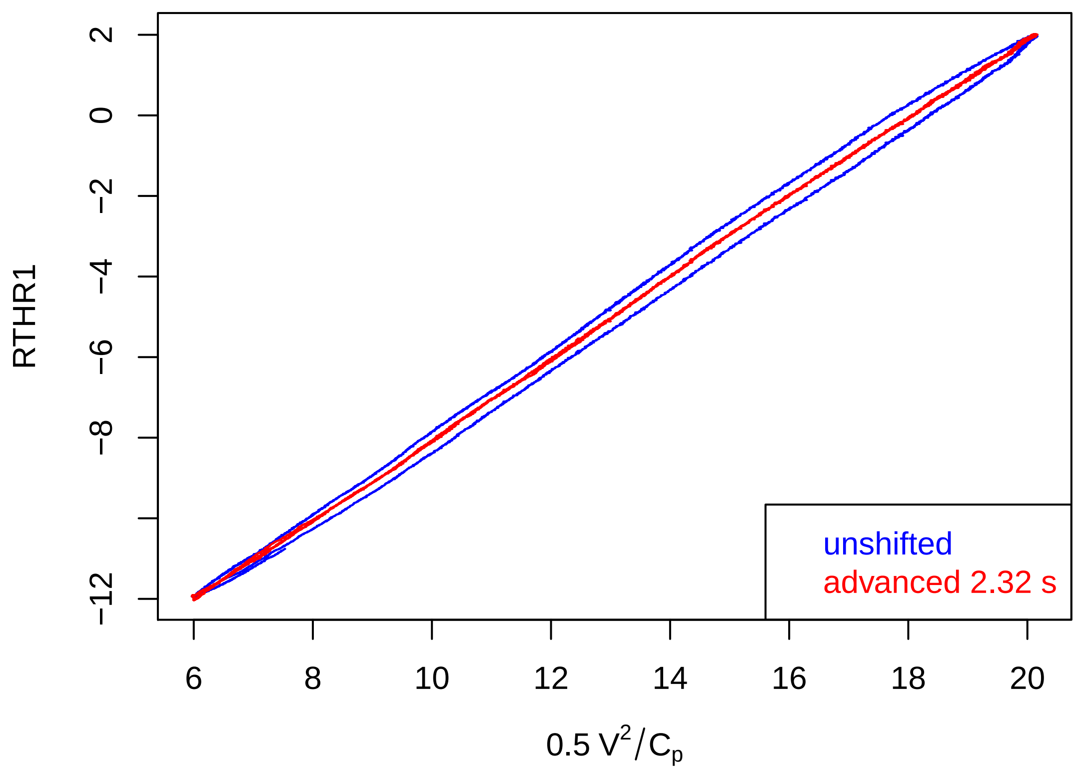

Figure 1: Recovery temperature RTH1 as a function of $V^2/(2C_p)$ for the values as originally measured (blue line) and after shifting by 2.32 s to minimize the difference between accelerating and decelerating branches of the speed run. Data from DEEPWAVE flight 15, 3:23:00–3:28:00 UTC.

For a good example, consider the speed run from the DEEPWAVE project flight 15 (3 July 2014), 3:23:00 – 3:28:00 UTC, shown in Fig. 1. The red line shows that the accelerating and decelerating portions of the speed run overlap almost completely, supporting that the time-lag correction applied (2.32 s or 58 25-Hz samples) is appropriate. The effect of shifting the measurements forward in time was to decrease the standard deviation of the striaight-line fit to the data in this figure from 0.24 to 0.06°C.

2 Speed runs

2.1 The data used

There are three calibrations involved in the system that measures temperature. First, the resistance of the sensing element as a function of temperature is determined by calibration in a temperature-controlled bath. The result is normally a good match to the Callendar-van-Dusen equation. Second, the A-to-D channels on the aircraft are calibrated by applying known resistances in place of the sensing elements and measuring the corresponding voltages. Thus with a measured voltage, one can determine the resistance of the sensing element and hence the temperature. In processing, this is represented by a quadratic polynomial that relates voltage to the sensed or 'recovery' temperature. Finally, the third calibration involved is to find the recovery factor used to calculate ambient temperature from recovery temperature. As of this date, we are using a report from Goodrich, which has led to (2) above. That report applies to the Rosemount 102 probe, but because the geometry of the HARCO probe is so similar was also use the same formula for the HARCO.

The purpose of this new study is to determine if the recovery factor can be determined or checked using flight data. "Speed runs" in which the airspeed is varied through the flight envelope of the aircraft in level flight are normally used for this purpose. Using such maneuvers, it should be possible to check the applicability of (2) by determining if the resulting ambient temperature given by (1) stays constant through the speed run.

Another possible approach is to consider the calibration coefficients for total temperature vs voltage and for a polynomial representation of the recovery factor all as variables to be fitted to minimize the differences between measured temperature differences and those expected from integration of the hydrostatic equation. This general approach was used in previous work to check that the coefficients were all reasonable, but here it will be explored if further fits can help constrain the recovery factor rather than just check if it is reasonable.

Toward this end, a search was conducted for all speed runs that could be found in past GV projects. An R script was used to search for appropriate variations in airspeed while the altitude remained constant, and that script produced the following set of 84 speed runs, included here in case there might be further use for this information.

PROJECT	start time (UTC)	end time (UTC)	low speed (m/s	high speed (m/s)
CONTRASTrf08.nc	82842	83744	129.422882	217.075256
CONTRASTrf08.nc	91651	92702	162.610886	253.096359
CONTRASTrf11.nc	42258	42500	169.818695	211.224686
DC3ff03.nc	160443	161051	170.144104	251.482666
DC3rf04.nc	203025	203313	139.280212	196.661362
DC3rf06.nc	205416	205632	143.778076	183.88797
DC3rf09.nc	1042	2105	185.569855	257.072937
DC3rf11.nc	192811	193033	171.655777	212.770844
DC3rf11.nc	192811	193034	169.86084	211.360291

PROJECT	start time (UTC)	end time (UTC)	low speed (m/s	high speed (m/s)
DC3rf15.nc	225820	230217	168.484116	208.525482
DC3rf17.nc	210522	210939	137.137161	186.421295
DC3rf22.nc	174858	175523	130.006897	215.78569
DC3-TESTrf01.nc	204038	204540	202.092758	259.001953
DC3-TESTrf01.nc	205506	205939	201.846298	257.073669
DC3-TESTrf02.nc	205351	205628	146.182526	196.892273
DC3-TESTrf04.nc	223023	223318	177.592514	239.396561
DC3-TESTrf04.nc	224447	224745	177.561554	238.508972
DEEPWAVErf06.nc	85253	85531	179.322189	229.620926
DEEPWAVErf10.nc	110226	110458	173.312851	236.500259
DEEPWAVErf15.nc	32048	32924	110.085167	201.115402
DEEPWAVErf15.nc	41449	42329	127.907669	220.444107
DEEPWAVErf15.nc	50045	51058	157.124557	244.054611
DEEPWAVErf20.nc	40250	40507	164.675598	204.97168
HCRTESTrf03.nc	203154	204142	126.6026	203.430801
HEFT10_ff01.nc	40210	40443	133.950653	183.21167
HEFT10_tf05.nc	192608	193158	110.08165	181.241989
HEFT10_tf05.nc	194731	195722	139.736877	218.432129
HIPPO-1rf02.nc	20435	20636	183.094238	227.701263
HIPPO-1rf11.nc	180722	180951	192.032944	237.40596
HIPPO-3rf08.nc	2612	2834	202.151047	242.238861
HIPPO-3rf10.nc	224424	224720	162.311966	221.918747
HIPPO-3tf01.nc	184447	184748	110.580902	166.816666
HIPPO-3tf02.nc	190547	191138	151.308929	243.920578
HIPPO-4rf02.nc	184014	184218	220.547546	267.082062
HIPPO-4rf02.nc	185213	185357	110.177368	196.568939
HIPPO-4rf02.nc	210749	210944	222.694214	270.595032
HIPPO-4rf02.nc	231904	232228	144.795227	206.885941
HIPPO-4rf02.nc	25735	30154	140.330765	222.947357
HIPPO-4rf12.nc	201320	201546	171.849274	230.06723
HIPPO-5rf01.nc	190029	191425	202.326447	253.88385
HIPPO-5rf02.nc	163045	163307	147.367447	196.610107
HIPPO-5rf03.nc	191115	191326	132.636002	179.867935
IDEAS-4rf01.nc	203131	203346	176.74086	233.033081
IDEAS-4rf04.nc	175852	180157	138.343262	193.757233
IDEAS-4rf04.nc	194433	195241	130.364929	213.857697
IDEAS-4rf04.nc	200012	201109	180.71907	255.520874
IDEAS-4rf04.nc	201910	202833	145.308716	234.262146
IDEAS-4rf05.nc	172703	172944	170.836182	224.205368
IDEAS-4rf05.nc	211448	212030	110.051697	158.519424

PROJECT	start time (UTC)	end time (UTC)	low speed (m/s	high speed (m/s)
IDEAS-4rf06.nc	181550	181830	168.659332	223.515488
IDEAS-4rf06.nc	183959	184220	164.784454	206.19104
IDEAS-4rf07.nc	192312	192932	207.630142	253.947754
IDEAS-4rf08.nc	1351	1925	110.016251	152.924255
IDEAS-4rf08.nc	193458	193729	129.243881	176.570114
IDEAS-4rf08.nc	203642	204341	133.652863	209.316956
IDEAS-4rf08.nc	211447	212030	110.044678	159.280121
IDEAS-4rf10.nc	210920	211149	137.971451	186.348709
IDEAS-GV_rf05.nc	172703	172944	170.837067	224.20665
PREDICTff01.nc	192609	193328	142.619598	213.723999
PREDICTff01.nc	195540	200222	129.052399	195.964722
PREDICTff01.nc	202858	203702	116.423767	178.38913
PREDICTff01.nc	205058	205604	117.371231	178.609619
PREDICTff01.nc	205058	205604	117.414474	178.837814
PREDICTff01.nc	210401	211235	116.865746	179.074387
PREDICTrf23.nc	200356	200602	121.432022	165.286499
PREDICTtf02.nc	201213	201545	140.128494	186.457169
PREDICTtf02.nc	203532	203929	110.513382	155.347427
PREDICTtf02.nc	211649	211910	212.972504	253.014954
PREDICTtf02.nc	214942	215639	134.66835	243.455307
PREDICTtf04.nc	194302	194631	122.792404	211.899673
PREDICTtf04.nc	202952	203226	137.630081	188.510025
SAANGRIA_rf01.nc	221318	221733	199.829071	241.277359
SPRITE-IIrf07.nc	90135	90737	135.038452	241.103867
SPRITE-IIrf07.nc	91059	91650	126.445938	230.574677
TOREROrf07.nc	180205	180428	200.23497	245.094971
TOREROrf07.nc	185330	185509	177.085007	219.028503
TOREROrf08.nc	220110	220316	127.408226	188.016235
TOREROrf09.nc	170519	170745	179.24205	225.697205
TOREROrf09.nc	211821	212051	140.474152	195.22551
TOREROrf11.nc	204447	204705	119.84536	174.486572
TOREROrf11.nc	211854	212051	141.645859	191.521744
TORERO_rf14.nc	175426	175619	127.66349	174.010803
TOREROrf16.nc	142318	142506	163.494308	208.249069
TOREROrf16.nc	224902	225402	132.090012	190.37558

2.2 Analysis approach

From this set of speed runs, a subset was selected where conditions appeared to be constant and where the speed run included both acceleration and deceleration so that overlap between these portions of the maneuver could be used to check for uniformity. The latter, for example, excluded all the TORERO cases because they only increased or decreased in speed without including both. Also, measurements from the HARCO sensors were selected initially because that was the sensor most often used. Also, initially only the 'A' element was considered. The result was 21 speed runs that appeared to provide the best information on the HARCO 'A' sensor.

For each of these 21 speed runs, the recovery-temperature measurements from the HARCO 'A' sensor were plotted against $X = V^2/(2C_p)$ so that the recovery factor could be determined from the slope of the plotted values. Regression was used to determine the slope for the entire speed run and also for subsets from each speed run that fell into bins of width 0.1 in Mach number M centered at 0.4, 0.5, 0.6, ..., 0.9 so that the possible Mach-number dependence of the recovery factor could be studied:

$$T_r = T_a + \alpha(M)\frac{V^2}{2C_p} \tag{4}$$

or

$$\frac{dT_r}{dX} = \alpha(M) \ .$$

However, M and X are not independent so a fit to a simple expansion of α in terms of powers of $\log_{10}(M)$, as was used to represent the wind-tunnel data, will not give the recovery factor directly. Instead, a polynomial of the form $T_r = \sum a_n X^n$ was determined by fitting the speed-run data and the resulting polynomial was differentiated to find $\alpha(X)$. Because $M^2 = V^2/(\gamma R T_a) = 2C_v X/(R T_a)$ and the fit also gives T_a, the resulting function $\alpha(X)$ can be transformed to $\alpha'(M) = \alpha(X = M^2 R T_a/(2C_v)$.

This suggests a way to combine the 21 good speed runs to obtain a composite recovery factor:

1. For each speed run, fit a 4th-order polynomial in X to $T_r(X)$.
2. From the derivative of that polynomial with respect to X, determine the recovery factor $\alpha(X)$.
3. For each point in the time series for the speed run, find corresponding values of X, α, and M. Add these to the data.frame containing the valid speed runs.
4. For the composite data.frame, find average values of α in bins of M, with standard deviations.
5. Fit the desired functional form for $\alpha(M)$ to these averages, in a weighted fit considering the standard deviations.

3 Results

3.1 The HARCO sensors

The result of doing this is shown in Fig. 2. The plot shows that, contrary to the representation of the recovery factor in the Goodrich technical note (for the Rosemount probe), there is no evidence of

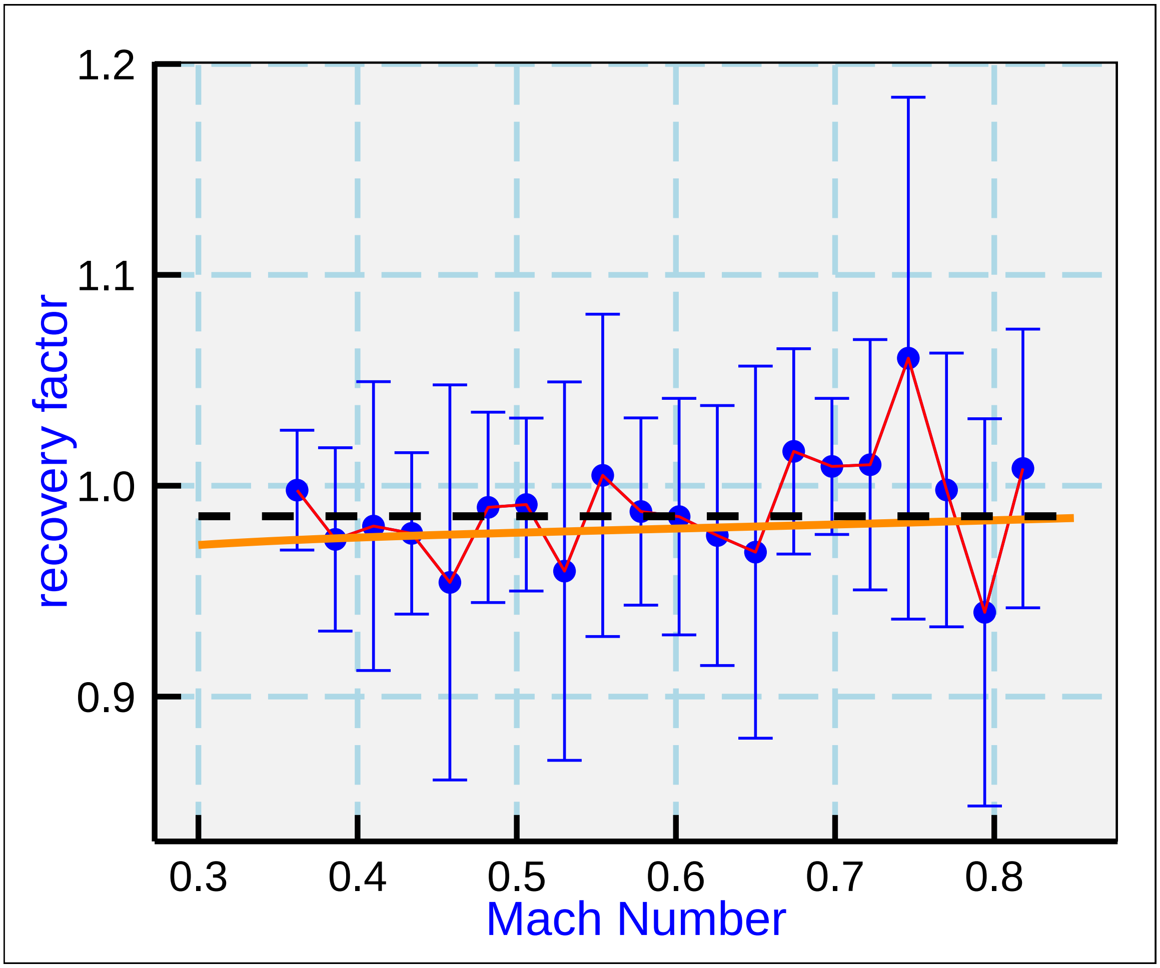

Figure 2: Mean values and standard deviations of the recovery factor after binning in Mach Number.

any Mach-number dependence in these measurements. The mean value for the recovery factor for all of these speed runs in 0.986. If the 21 speed runs are considered independent determinations of the recovery factor, the standard deviation in the mean would be about 0.016. This value is plotted as the black dashed line in Fig. 2. For comparison, the Mach-number dependence previously obtained from the Goodrich report is plotted also on Fig. 2, as the orange line. It is consistent with the measurements also, so with this check it should be justified to use this recovery-factor function for the HARCO sensors.

This analysis chain was also repeated for the 'B' HARCO elements, the second element in the housing, on the chance that this element might have a different recovery factor. The results are shown in Fig. 3. The mean recovery factor for these elements is a little lower, 0.969, and in comparison to the standard Mach-dependent formula (shown as the orange line) it is about 0.015 smaller over the primary flight envelope (about Mach 0.5–0.8). Because there is no evidence of any dependence on Mach number, it appears justified for this element to use a constant value, 0.969,

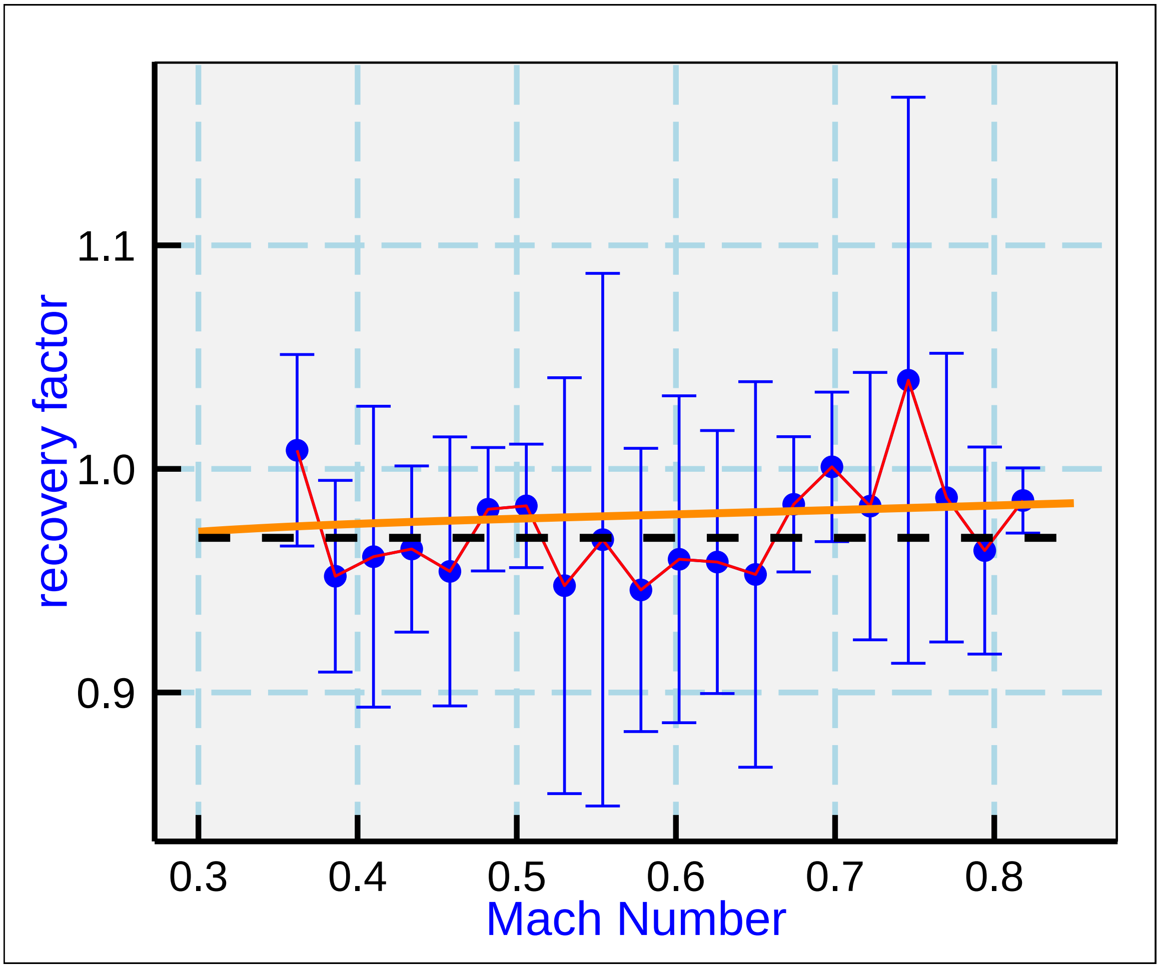

Figure 3: Mean values and standard deviations of the recovery factor after binning in Mach Number, for the HARCO-B elements.

for the recovery factor.

3.2 The heated Rosemount sensors

There have been fewer high-quality speed runs with the Rosemount dual-element heated sensors. The seven that were identified will be used here. [1]

The results are shown in Fig. 4. The mean value for element-A of the heated Rosemount probes was 0.958, significantly below the value for the HARCO probes or the standard recovery-factor formula. This seems to be enough of a discrepancy that a different recovery factor should be used

[1] There may be some more, especially from Progressive Science where there are an additional five good speed runs, but I haven't yet confirmed that the sensors were Rosemount probes.

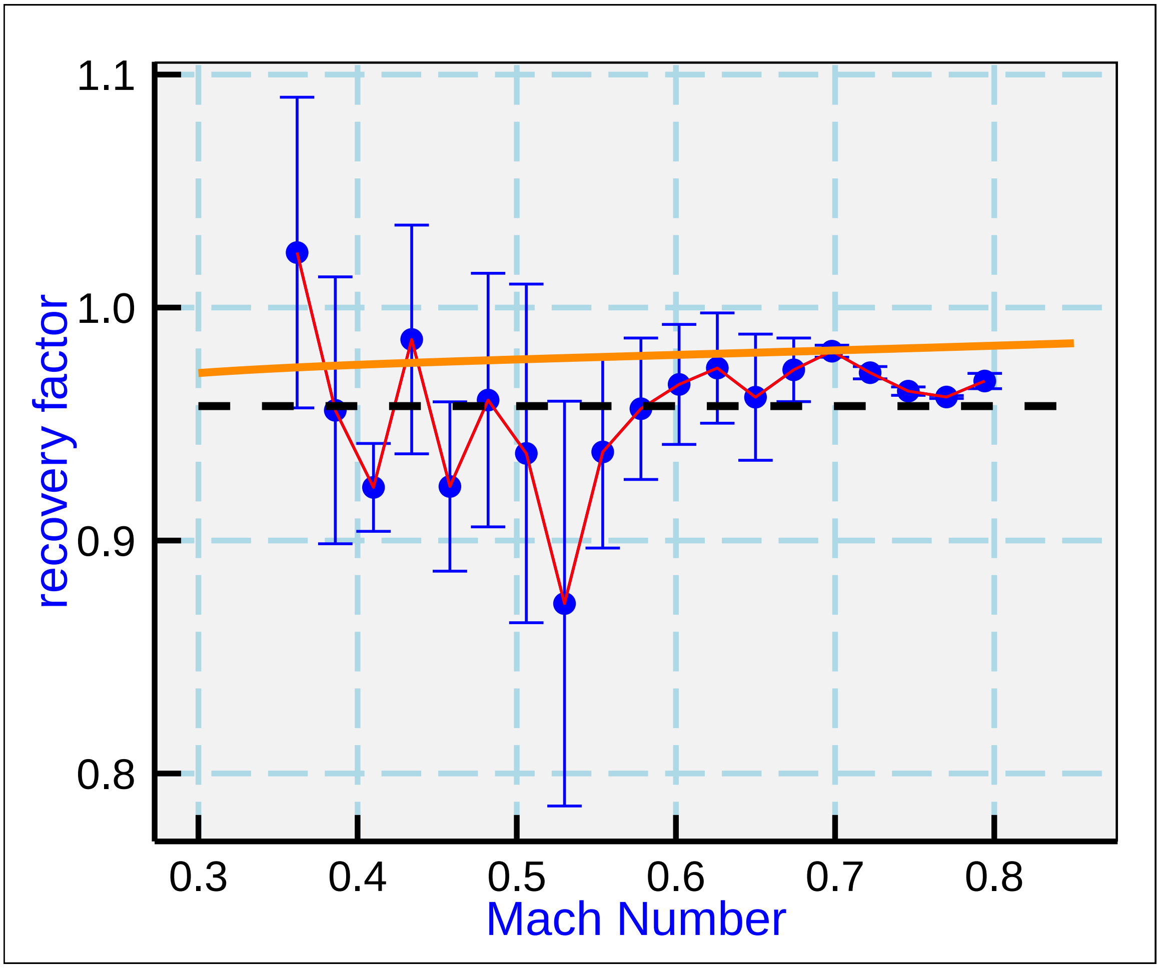

Figure 4: Mean values and standard deviations of the recovery factor after binning in Mach Number, for the Rosemount-A elements.

for the Rosemount probes, and there doesn't seem to be any reason to use a dependence on Mach number, so the value 0.958 appears to be appropriate.

For element B of the heated Rosemount probes, the corresponding value is almost identical, 0.957, as shown in Fig. 5. The same recovery factor therefore seems appropriate for both elements of the heated Rosemount probe.

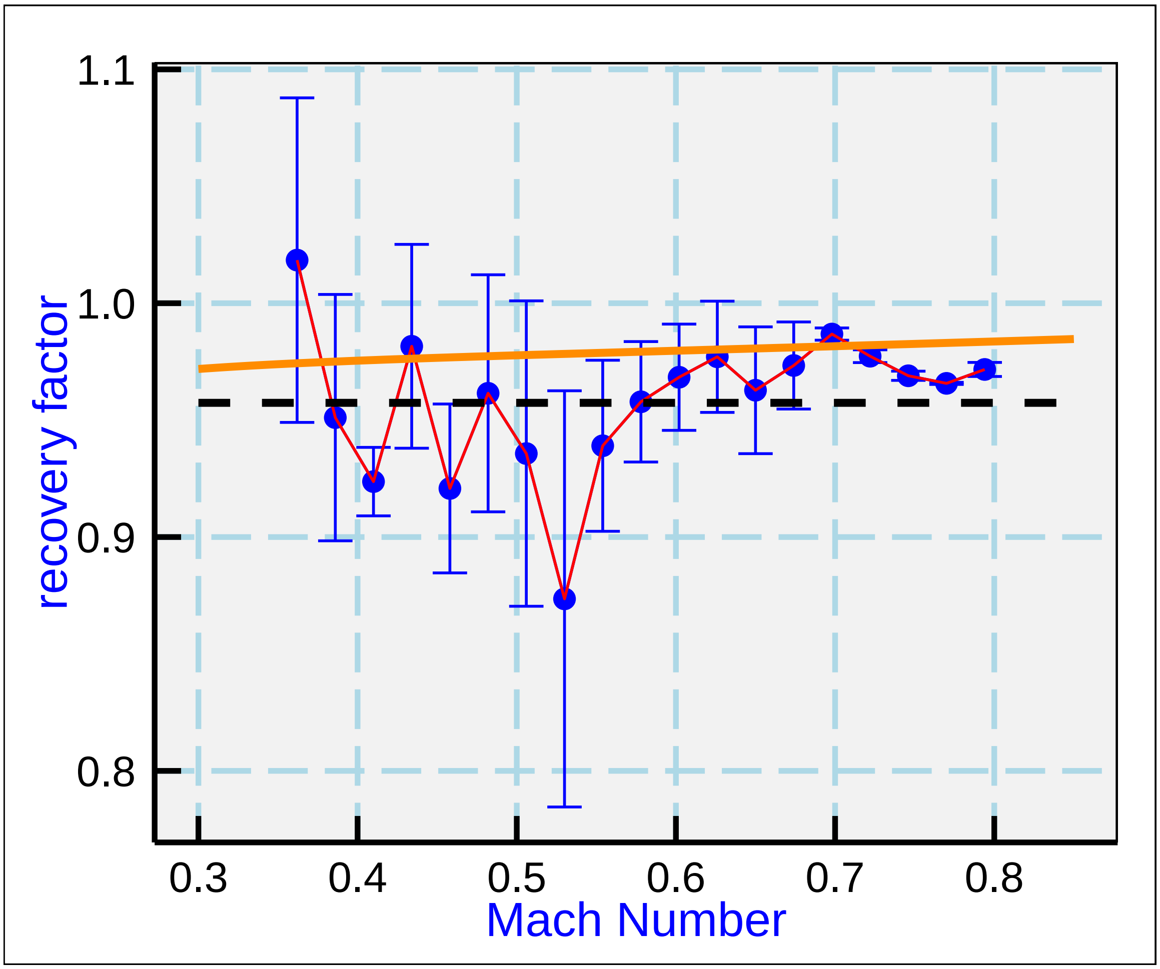

Figure 5: Mean values and standard deviations of the recovery factor after binning in Mach Number, for the Rosemount-B elements.

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Reproducibility:

PROJECT:	RecoveryFactorStudy
ARCHIVE PACKAGE:	RecoveryFactorStudy.zip
CONTAINS:	attachment list below
PROGRAM:	RecoveryFactorStudy.Rnw
ORIGINAL DATA:	various, mostly in /scr/raf/Prod_Data
GIT:	git@github.com:WilliamCooper/Reprocessing/RecoveryFactorStudy.git

Attachments: RecoveryFactorStudy.Rnw
RecoveryFactorStudy.pdf
RecoveryFactorStudy.Rdata
SessionInfo